MATH1520AB 2021-22 Tutorial 6 (week 10)

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1. Evaluate the following indefinite integrals using substitution.

   (a) $\int 4\sqrt{5+9t} + 12(5+9t)^7 \, dt$

   (b) $\int \frac{4w+3}{4w^2+6w-1} \, dw$

   Answer.

   (a) Let $u = 5 + 9t$, thus we have $du = 9dt$ .Hence we can get $\int 4\sqrt{5+9t} + 12(5+9t)^7 \, dt = \int \left[4u^{\frac{1}{2}} + 12u^7\right]\left(\frac{1}{9}\right) du = \frac{1}{9}\left[\frac{8}{3}u^{\frac{3}{2}} + \frac{3}{2}u^8\right] + c = \frac{1}{9}\left[\frac{8}{3}(5+9t)^{\frac{3}{2}} + \frac{3}{2}(5+9t)^8\right] + c.$

   (b) Let $u = 4w^2 + 6w - 1$, thus we have $du = (8w+6)dw$. Hence we can get $4w + 3 = \frac{du}{2dt}$. So $\int \frac{4w+3}{4w^2+6w-1} \, dw = \int \frac{4w+3}{4w^2+6w-1} \, dw = \frac{1}{2}\int \frac{1}{u} \, du = \frac{1}{2}\ln|u| + c.$

2. Evaluate $\int \mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) \, dz.$

   Answer. $\int \mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) \, dz = \frac{1}{2}\mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) + \frac{1}{8}\int \mathbf{e}^{2z} \sin\left(\frac{1}{4}z\right) \, dz = \frac{1}{2}\mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) + \frac{1}{8}\left[\frac{1}{2}\mathbf{e}^{2z} \sin\left(\frac{1}{4}z\right) - \frac{1}{8}\int \mathbf{e}^{2z}\right.$ $\frac{1}{2}\mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) + \frac{1}{16}\mathbf{e}^{2z} \sin\left(\frac{1}{4}z\right) - \frac{1}{64}\int \mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) \, dz.$ Hence we can get the final answer is $\int \mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) \, dz = \frac{32}{65}\mathbf{e}^{2z} \cos\left(\frac{1}{4}z\right) + \frac{4}{65}\mathbf{e}^{2z} \sin\left(\frac{1}{4}z\right) + c$